## Problems

29. Organize and Plan Use Newton's law of gravitation (Eq. 9.1) to find the approximate attractive force due to gravity between the two football players.
Known: $r=25.0 \mathrm{~m}, m_{1}=m_{2}=115 \mathrm{~kg}$.
Solve Newton's law of gravitation gives [Eq. 1]

$$
F=\frac{G m_{1} m_{2}}{r^{2}}=\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~kg}^{2} / \mathrm{m}^{2}\right)(115 \mathrm{~kg})^{2}}{(25.0 \mathrm{~m})^{2}}=1.41 \times 10^{-9} \mathrm{~N}=1.41 \mathrm{nN}
$$

where the symbol " nN " represents nano-newtons.
REFLECT Note that this is an approximate calculation since the football players are not far enough apart relative to their size to be considered point particles. If they were, say, 25 km apart, the calculation would be much more accurate.
33. Organize and Plan Newton's law of gravitation works for protons and electrons as well as for heavenly bodies. The mass of the proton is [Eq. 1] $m_{p}=1.67 \times 10^{-27} \mathrm{~kg}$, and of the electron is [Eq. 2]
$m_{\mathrm{e}^{-}}=9.11 \times 10^{-31} \mathrm{~kg}$
Known: $r=5.29 \times 10^{-11} \mathrm{~m}$.
Solve Using Newton's law of gravitation, the force between the electron and proton is [Eq. 3]

$$
\begin{aligned}
& F=\frac{G m_{p} m_{\mathrm{e}^{-}}}{r^{2}} \\
& F=\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~kg}^{2} / \mathrm{m}^{2}\right)\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(9.11 \times 10^{-31} \mathrm{~kg}\right)}{\left(5.29 \times 10^{-11} \mathrm{~m}\right)^{2}}=3.63 \times 10^{-47} \mathrm{~N}
\end{aligned}
$$

Reflect The electromagnetic force is some $10^{36}$ times stronger than the force due to gravity. This means the electromagnetic force between two protons a light-year apart would be approximately the same as the gravitational force between two protons a centimeter apart.
36. Organize and Plan Use Newton's second law and Newton's law of gravitation to find the acceleration at the asteroid's surface. Calculate the total mass $M$ of the asteroid for use in Newton's law of gravitation [Eq. 1].

$$
M=\rho V=\frac{4}{3} \pi r^{3} \rho
$$

where $\rho$ is the density.
Known: $\rho=3500 \mathrm{~kg} / \mathrm{m}^{3}, r=9.50 \mathrm{~km}$.
Solve Newton's law of gravitation gives the force on an object of mass $m$ as [Eq. 2]

$$
F=\frac{G M m}{r^{2}}
$$

Combining Eq. (2) with Newton’s second law, and using Eq. (1), gives [Eq. 3]

$$
\begin{aligned}
F & =m a=\frac{G M m}{r^{2}} \\
a & =\frac{4}{3} \pi r G \rho=\frac{4}{3} \pi\left(9.50 \times 10^{-3} \mathrm{~m}\right)\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(3500 \mathrm{~kg} / \mathrm{m}^{3}\right)=9.29 \times 10^{-3} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Reflect This is approximately 1000 times less than the acceleration due to gravity on the surface of the Earth.
43.Organize and Plan Refer to Fig. 9.8 (a) for the definition of the major (a) and minor (b) axes of an ellipse. Use Eq.
9.3 to find the eccentricity $e$. We are told that [Eq. 1] $a=2 b$.

Solve Using Eq. 9.3 and Eq. (1), the eccentricity is [Eq. 2]

$$
e=\sqrt{1-\left(\frac{b}{a}\right)^{2}}=\sqrt{1-\left(\frac{1}{2}\right)^{2}}=0.866
$$

Reflect The eccentricity is thus intermediate between that of the ellipses shown in Fig. 9.9.
49.Organize and Plan Use Kepler’s third law applied to circular orbits (Eq. 9.4). The radii of the satellites’ orbits are related by [Eq. 1] $R_{A}=2 R_{B}$.
Solve Applying Kepler's third law to the orbit of satellites A and B, and using Eq. (1), gives [Eq. 2]

$$
\begin{aligned}
& \frac{R_{B}^{3}}{T_{B}^{2}}=\frac{G M}{4 \pi^{2}}=\frac{R_{A}^{3}}{T_{A}^{2}} \\
& \frac{T_{B}}{T_{A}}=\sqrt{\frac{R_{B}^{3}}{R_{A}^{3}}}=\sqrt{\left(\frac{1}{2}\right)^{3}}=\sqrt{1 / 8} \approx 0.35
\end{aligned}
$$

Reflect Reducing the radius by a factor of 2 results in a factor of $\sim 3$ reduction in the period.
56. Organize and Plan Apply Eq. 9.5 to the proton-electron system. Obtain the necessary data from the Internet. Known: $m_{p}=1.67 \times 10^{-27} \mathrm{~kg}, m_{e}=9.12 \times 10^{-31} \mathrm{~kg}, r_{e-p}=5.29 \times 10^{-11} \mathrm{~m}$.
Solve (a) Using Eq. 9.5, the potential energy of the electron-proton system is [Eq. 1]

$$
U_{e-p}=-\frac{G m_{p} m_{e}}{r_{S-J}}=-\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(9.12 \times 10^{-31} \mathrm{~kg}\right)}{5.29 \times 10^{-11} \mathrm{~m}}=-1.92 \times 10^{-57} \mathrm{~J}
$$

(b) This is 39 orders of magnitude less than the atomic binding energy.
(c) If gravity were the only force acting on the electron-proton system, the radius of the orbit of the electron around its proton would be larger than the known universe!
Reflect This problem gives an idea of the relative strength of the electromagnetic force than holds an electron in its orbit about a proton relative to the gravitational force.
58. Organize and Plan Use conservation of mechanical energy to find the maximum height attainable by the rocket. The initial mechanical energy is [Eq. 1]

$$
E_{0}=\frac{1}{2} m_{\text {rocket }} v^{2}-\frac{G M_{E} m_{\text {rocket }}}{R_{E}}
$$

and the final mechanical energy at the maximum height $h$ is

$$
E_{f}=-\frac{G M_{E} m_{\text {rocket }}}{R_{E}+h}
$$

Known: $v=2200 \mathrm{~m} / \mathrm{s}, M_{E}=5.97 \times 10^{24} \mathrm{~kg}, R_{E}=6.37 \times 10^{6} \mathrm{~m}$ (Appendix E).
SOLVE Equating the initial and final mechanical energies and solving for $h$ gives

$$
\begin{aligned}
-\frac{G M_{E} m_{\text {rocket }}}{R_{E}+h} & =\frac{1}{2} m_{\text {rocket }} v^{2}-\frac{G M_{E} m_{\text {rocket }}}{R_{E}} \\
h & =\frac{G M_{E} R_{E}}{G M_{E}-(1 / 2) R_{E} v^{2}}-R_{E} \\
& =\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.97 \times 10^{24} \mathrm{~kg}\right)\left(6.37 \times 10^{6} \mathrm{~m}\right)}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.97 \times 10^{24} \mathrm{~kg}\right)-(1 / 2)\left(6.37 \times 10^{6} \mathrm{~m}\right)(2200 \mathrm{~m} / \mathrm{s})^{2}}-6.37 \times 10^{6} \mathrm{~m} \\
& =2.57 \times 10^{5} \mathrm{~m}
\end{aligned}
$$

Reflect In Problem 57, we found the velocity of a ball dropped from a height of $10^{4} \mathrm{~m}$ is $443 \mathrm{~m} / \mathrm{s}$, whereas from $10^{6} \mathrm{~m}$ it is $4,119 \mathrm{~m} / \mathrm{s}$. Since the result of $2.57 \times 10^{5} \mathrm{~m}$ is intermediary between these two velocities, it is consistent since the final height reached by the rocket is intermediary between the two heights from which that ball was dropped.
68.Organize and Plan The escape speed from the surface of Mars can be found using Eq. 9.6, replacing the Earth's mass and radius with the mass and radius of Mars.
Known: $M_{\text {Mars }}=6.42 \times 10^{23} \mathrm{~kg}, R_{\text {Mars }}=3.37 \times 10^{6} \mathrm{~m}$.
Solve The escape speed from the surface of Mars is [Eq. 1]

$$
v_{\mathrm{esc}}=\sqrt{\frac{2 G M_{\text {Mars }}}{R_{\text {Mars }}}}=\sqrt{\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(6.42 \times 10^{23} \mathrm{~kg}\right)}{3.37 \times 10^{6} \mathrm{~m}}}=5.04 \mathrm{~km} / \mathrm{s}
$$

Reflect This is less than half the escape speed from the Earth's surface, which is $11.2 \mathrm{~km} / \mathrm{s}$.
76.Organize and Plan From Ex. 9.13, the radius of a geosynchronous orbit is $R=4.22 \times 10^{7} \mathrm{~m}$. Use Eq. 9.7 to find the total mechanical energy of the satellite on the surface of the Earth and in geosynchronous orbit. Take the difference of these values to find the energy needed to place the satellite in geosynchronous orbit.
Known: $R_{\mathrm{E}}=6.37 \times 10^{6} \mathrm{~m} ; \mathrm{m}=1.0 \mathrm{~kg} ; R=4.22 \times 10^{7} \mathrm{~m} ; M_{\mathrm{E}}=5.97 \times 10^{24} \mathrm{~kg}$ (Appendix E).
Solve Using Eq. 9.7, The energy of an satellite at rest on the Earth's surface is

$$
E_{0}=-\frac{G M_{\mathrm{E}} m}{R_{\mathrm{E}}}
$$

and when it's in orbit, the total mechanical energy is [Eq. 2]

$$
E_{\mathrm{f}}=-\frac{G M_{\mathrm{E}} m}{2 R}
$$

Subtracting the initial energy from the final energy gives the energy needed to put the satellite into orbit, which is [Eq. 3]

$$
\begin{aligned}
E & =E_{\mathrm{f}}-E_{0}=\frac{G M_{\mathrm{E}} m}{}\left(\frac{1}{R_{\mathrm{E}}}-\frac{1}{2 R}\right) \\
& =\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.97 \times 10^{24} \mathrm{~kg}\right)(1.0 \mathrm{~kg})}{}\left(\frac{1}{6.37 \times 10^{6} \mathrm{~m}}-\frac{1}{2 .\left(4.22 \times 10^{7}\right)}\right) \\
& =5.77 \times 10^{7} \mathrm{~J}
\end{aligned}
$$

REFLECT Even though the satellite's mass is 1.0 kg , it is still a good idea to put it into Eq. (3) to keep the units are correct and to keep track of the significant figures.
82. Organize and Plan A light-year is the distance traveled by light in one year. The distance to the center of the galaxy is 25,000 light-years, which in meters is [Eq. 1]

$$
R=25,000 \text { years } \times c=25,000 \text { years }\left(\frac{365 \mathrm{~d}}{1 \text { year }}\right)\left(\frac{24 \mathrm{~h}}{1 \text { day }}\right)\left(\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right)\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right) \times\left(3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)=2.37 \times 10^{20} \mathrm{~m}
$$

Convert all quantities to SI units for the calculation.
Known: $v=230 \mathrm{~km} / \mathrm{s}$.
Solve To find the orbital period, recall that the orbital speed is the orbit's circumference divided by its period [Eq. 2].

$$
\begin{aligned}
& v=\frac{2 \pi R}{T} \\
& T=\frac{2 \pi R}{v}=\frac{2 \pi\left(2.37 \times 10^{20} \mathrm{~m}\right)}{2.30 \times 10^{5} \mathrm{~m} / \mathrm{s}}=6.47 \times 10^{15} \mathrm{~s}=2.05 \times 10^{8} \text { years }
\end{aligned}
$$

To find the mass of the galactic center, use Kepler's third law applied to circular orbits [Eq. 3]; see (Eq. 9.4).

$$
\begin{aligned}
\frac{R^{3}}{T^{2}} & =\frac{G M}{4 \pi^{2}} \\
M & =\frac{4 \pi^{2} R^{3}}{G T^{2}}=\frac{4 \pi^{2}\left(2.37 \times 10^{20} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(6.47 \times 10^{15} \mathrm{~s}\right)^{2}}=1.88 \times 10^{41} \mathrm{~kg}
\end{aligned}
$$

Reflect The mass of the galactic center is approximately that of 100 billion Suns.

